**Homework 4 Questions**

# **Problem 1**: (15 points) Answer each part TRUE or FALSE (Big ).

* True: As , any input we want, we can always find a constant such that . For example, consider and ,

*for all*

* False: Since grows faster than , dominates the growth of . Therefore, cannot be bounded by , making false.
* True: According to the book, when the symbol occurs in an exponent, as in the expression , the exponent dominates the expression, thus representing an upper bound of . This means that is an upper bound for , and , we have . Hence, we can conclude that .

# **Problem 2**: (15 points) Answer each part TRUE or FALSE (Small ).

* False: grows at the same rate as , which means that as approaches infinity, the ratio

remains constant when the small definition requires that as approaches infinity, the ratio approaches zero. This would indicate that when the definition requires for any real number , there exists a real number for all . However, this can be violated with for any value of .

* True: grows slower than , which means that as approaches infinity, the ratio

ratio approaches zero, which satisfies the small definition.

* False: grows slower than , which means that as approaches infinity, the ratio approaches infinity, not zero as required by the small definition.

# **Problem 3**: (10 points) Is the following pair of numbers relatively prime? Show the calculations that led to your conclusion.

## and

* Yes, this pair of values are relatively prime. Since Two numbers are relatively prime if 1 is the largest integer that evenly divides them both. We can calculate this using the Euclidean algorithm to find . Our Process is as follows:
  1. Using Division Algorithm, find q and r to write a=bq+r
  2. Do step 1 again, but now use as your new , and use as your new
  3. Stop when your . Your from the last step is your final answer.
* Proof:

#### Using Division Algorithm, find and to write

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* + By definition, two numbers are relatively prime if is the largest integer that evenly divides them both. Thus, in order to prove that and are relatively prime, I calculated their greatest common divisor (gcd) using the Euclidean algorithm, and obtained . As such, I have successfully prove that and have no common divisors other than , and therefore they are relatively prime.

# **Problem 4**: (10 points) Is the following formula satisfiable? Why?

First, lets review the Boolean truth values

|  |  |  |  |
| --- | --- | --- | --- |
| and conjunction (intersection) | | or disjunction (union) | |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
| not *negation (complement)* | | | |
|  | |  | |
|  | |  | |

According to our textbook, a Boolean formula is satisfiable if some assignment of s and s to the variables makes the formula evaluate to .

Therefore, to test the satisfiability of this equation, we can try different combinations of and to check if any of them evaluate to the value of . There are two possible scenarios to consider: and , or and . It is not necessary to test the scenarios where and have the same value since they are represented by the same symbol.

## Scenario 1: and

Scenario 1 evalutates to 0 and, as such, we know that and can not be the correct values to prove this formulas satisfiability.

## Scenario 2: and

After testing Scenario 2, I have shown that it also evaluates to . Therefore, none of the possible assignments of s and s to the variables can make the formula evaluate to 1. This proves that the Boolean equation is not satisfiable.

# **Problem 5**: (10 points) What is P-Problem? What is NP-Problem? What is NP-Complete problem?

A P-Problem is a problem that is efficiently decidable in polynomial, such as , time on a deterministic single-tape Turing machine. These problems are considered "tractable" and the class of problems that are realistically solvable can computed using the function:

*“union of all polynomial time functions”*

An NP-Problem stands for "nondeterministic polynomial time" and it is a decision problem that can be solved by a non-deterministic Turing machine in polynomial time. In other words, given a potential solution to an NP problem, it can be verified to be correct or incorrect in polynomial time. However, finding a solution may require a non-polynomial amount of time.

NP-Complete refers to a class of problems whose “individual complexity is related to that of the entire class.” In other words, if an efficient, polynomial time algorithm exists for solving a problem within the class, then an efficient algorithm exists for solving all problems in NP. An example of an NP-Complete problem includes the Boolean satisfiability problem.

# **Problem 6**: (20 points) Show that NP is closed under union

(Hint: For any two languages L1 and L2, let M1 and M2 be the NTM that decides them in polynomial time. Construct a NTM M’ that decided L1UL2 in polynomial time.)

To show that NP is closed under union, I need to prove that for any two languages and in NP, their union is also in NP. In order to do this, I will begin by constructing a non-deterministic Turing machine (NTM) that decides in polynomial time.